PHY 540 Statistical Mechanics (Fall 2021) Homework 9
Posted: Tuesday Nov 16
Due: Tuesday Nov 30

9.1 Fluctuations of particle number(15)

Calculate the fluctuations of the occupancy numbers N_k of a single state k in thermodynamic equilibrium in the cases of classical particles, fermions, and bosons, and express them as functions of $\langle N_k \rangle$. Compare the results and discuss the limit in which they become equal.

9.2 Spin fluctuations(15)

Consider a system of N non-interacting Ising spins $s_i = \pm 1$,

$$\mathcal{H} = -h \sum_i s_i$$

and calculate the r.m.s. fluctuation of the total magnetization $m = \sum_i s_i$.

9.3 Energy fluctuation (20)

Calculate the r.m.s. fluctuation of the energy δE for a three-spin Ising model,

$$\mathcal{H} = -J(s_1 s_2 + s_1 s_3 + s_2 s_3)\,.$$

9.4 Harmonic oscillator in thermodynamic equlibrium(25)

Consider a harmonic oscillator with frequency $\omega_0 = \sqrt{k/m}$ and small damping factor η in thermal equlibrium with its environment

$$m\ddot{q} + \eta\dot{q} + kq = \Delta F_{env}$$

Using the relations between the damping factor η and the spectral density S_F, find the autocorrelation of the oscillator coordinate

$$K_q(\tau) = \langle q(t)q(t+\tau) \rangle\,.$$

9.5 Shot noise(25)

Calculate the spectral density of fluctuations of the current of charged carriers q flowing through a junction shown in the figure. Assume that the charged carriers pass the junction randomly and independent of each other, and that the average current is equal to $\bar{I}(t)$.