8.1   Equilibrium of ideal gas with traps

(a) To compute the entropy associated with the traps, count the number of microstates (statistical weight) for N_b filled traps out of N_T total, and use the Boltzmann's formula to get $S_T(N_b)$. Note that it does not depend on the actual temperature, because it depends only on the "disorder" associated with the traps, i.e. lack of information about which particular traps are occupied. Then, find the free energy (same as the Gibbs potential since there is no volume) $F(T, N_b) = E_T(N_b) - TS_T(N_b) \equiv G(T, N_b)$; the chemical potential can be determined from $\left(\frac{\partial G}{\partial N_b}\right)_{T, N_T}$.

    An alternative way to calculate the entropy is to use the Gibbs distribution and compute the partition function at some *fictitious* temperature T^* by treating each trap as a two-level system: empty with energy 0 or filled with energy $(-\Delta)$. To get $S_T(N_b)$, eliminate the fictitious temperature T^* from $S_T(T^*) = -\left(\frac{\partial F}{\partial T^*}\right)$ and the number of filled traps $\langle N_b \rangle(T^*)$.

    Note that the free energy and the Gibbs potential above will depend on the *actual* temperature T. The number of filled traps in the equilibrium with the gas depends not only on the temperature, but also on the number of molecules *available* to be trapped (e.g., the density of the gas). The fictitious temperature T^* above serves only as a tool to compute the traps' entropy function $S_T(N_b)$.

(b) Since there is no volume associated with the traps, the free energy and the Gibbs potential are the same. The chemical potential can be found as their change when one more trap is filled (it is generally assumed that $N \gg 1$, so derivatives can be calculated w.r.t. N).

(c) Write and solve the "chemical" equilibrium equation between the gas and the trapped molecules.

8.2   Mean-field theory of the Ising model

One way is to compare the predictions from the Ising model and the mean-field theory, e.g., the critical temperature, magnetization, susceptibility, etc. The theories should match near the critical point because their critical exponents match.

8.3   First-order phase transition

(a) When sketching $\phi(\eta)$, pay attention to its local minima. Each local minimum corresponds to a different phase. It may be helpful to use some plotting software.

(b) Since it is said that the local minima cannot be maintained, the system will switch the phase immediately when the local minimum describing the current phase becomes larger than the other local minimum; the temperature will be the transition temperature.

(c) The latent heat can be computed from the difference between the entropies in each phase. Each phase corresponds to a minimum of $\phi(\eta)$, and the entropy can be calculated as a temperature derivative of ϕ such that $\eta(\tau)$ remains the local minimum.