PHY 540 Statistical Mechanics (Fall 2021) Homework 7
Posted: Tuesday Nov 2
Due: Tuesday Nov 9

7.1 3D→2D condensation of ideal gas

A classical ideal gas of N particles is placed into a container of volume V and wall surface area A, on which the particles may condense into liquid. This liquid phase can be treated as a 2D ideal gas with binding energy Δ per particle, i.e. particle's energies in the gas and in the liquid are

$$H_{\text{gas}} = \frac{p_{3D}^2}{2m}\,, \quad H_{\text{liquid}} = \frac{p_{2D}^2}{2m} - \Delta\,.$$

(a) Find the relation between the pressure of the saturated vapor (the 3D gas pressure in equilibrium with the 2D liquid) and the surface density of condensed molecules $n_{\text{cond}} = N_{\text{cond}}/A$.
(b) Now assume that the surface density of condensed particles is maintained constant, $N_{\text{cond}}/A = \text{const}$. Calculate the latent heat of evaporation $l = L/N$ per particle (*hint: $l \neq \Delta$; use the Clapeyron-Clausius equation*).

7.2 Boiling temperature of water

Since the air pressure decreases with elevation, so does the boiling temperature of water T_B. Use the barometric formula $p(h) = p(0)\exp(-mgh/T_{\text{air}})$ and the Clapeyron-Clausius equation to estimate the difference of the water boiling temperature at the sea level and at 1-mile elevation (e.g. in Denver). Assume that the air temperature $T_{\text{air}} = 290\,\text{K}$ is uniform, as well as its molar mass $\mu_{\text{air}} = 29\,\text{g/mol}$. Assume also that the latent heat of water evaporation is independent of pressure $L \approx 2.2 \times 10^3 \text{J/g} \approx 40 \times 10^3 \text{J/mol}$. If you need the equation of state for the water vapor, treat it as an ideal gas.

7.3 3-spin Ising model

Consider the Hamiltonian for three classical (Ising) spins ,

$$\mathcal{H} = -J(s_1 s_2 + s_1 s_3 + s_2 s_3) - H(s_1 + s_2 + s_3)$$

where $s_i = \pm 1$, the ferromagnetic coupling $J > 0$, and H is the magnetic field. Enumerate the states of the system (energies and degeneracies) and calculate the partition function. What is the magnetization of the system in the absense of magnetic field? Calculate the magnetic field susceptibility and the heat capacity in the high-temperature limit.