

5.1 Electron bands in a semiconductor

(a) Treat electrons and holes as independent ideal gases in their respective energy bands; use the relation between the chemical potential and the density (don't forget the spin degeneracy). Also, you can take the band gap into account when computing chemical potentials so that both gases have the same energy baseline (which will also be the baseline for the chemical potential).

(b) Write the equation for chemical potentials in the process $e + h \rightarrow$ nothing that would imply particle balance (make sure the energy in both gases is measured from the same baseline, or “released” in the chemical potential equation). Combine this equation with the $n_e = n_h$ condition for the undoped semiconductor.

5.2 Thermocouple

As discussed in the lecture, the chemical potential of degenerate Fermi gas depends on the temperature. (Overall electron density has to stay close to constant so that the metal is electrically neutral at any point). Varying chemical potential induces diffusion that tries to reduce the total Gibbs potential, until the redistributed charges create electric potential counteracting this diffusion; this potential is equal to thermo-EMF. Alternatively, one can apply electric potential of the same magnitude to counteract this diffusion beforehand and prevent any redistribution of charges; this logic leads to the same answer.

Thus, the magnitude of thermo-EMF can be deduced from the differences of the chemical potential summed around a closed loop (similar to the 2nd Kirchhoff's law). Note that the chemical potential has to be equal in metals at the point of contact, otherwise electrons will migrate into the one where it is lower.

5.3 Degenerate relativistic Fermi gas

All calculations can be performed as if $T = 0$. Calculate the Fermi energy and the total (internal) energy from the density the same way as for the nonrelativistic degenerate Fermi gas discussed in the lecture (the only difference is the energy vs. momentum dependence). For the degenerate gas, entropy is zero, so the pressure can be found as $p = \left(\frac{\partial E}{\partial V}\right)_S \stackrel{T=0}{=} \left(\frac{\partial E}{\partial V}\right)_{T=0}$. Alternatively, calculate the grand potential for $T = 0$ and then pressure as $p(T = 0) = -\Omega(T = 0)/V$.

5.4 Paramagnetism of a degenerate Fermi gas

(a) Treat the gas as a mixture of spin-up and spin-down components that are in equilibrium. The chemical potential (\approx Fermi energy ε_F for $T \rightarrow 0$) will include the spin-in-magnetic field energy, which leads to different densities of spin-up and -down and hence net magnetization.

(b) For ultrarelativistic gas, the calculation of the Fermi energy is different, which leads to different net magnetization and susceptibility.