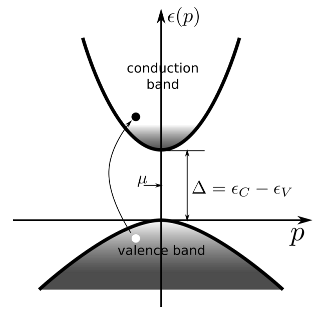
5.1 Electron bands in a semiconductor

A semiconductor may be described by a valence and a conduction electron energy bands, which are completely filled and empty, respectively, at $T = 0$. At $T > 0$, electrons may be excited and tunnel into the conduction band leaving behind a "hole" in the valence band. Taking the top of the valence band as zero, the electron energies in these bands are



$$\varepsilon_V(p) = -\frac{p^2}{2m_V}\,, \quad \varepsilon_C(p) = \frac{p^2}{2m_C} + \Delta$$

where Δ is the band gap, in which there are no electron energy levels, and $m_{V,C}$ are the electron effective masses.

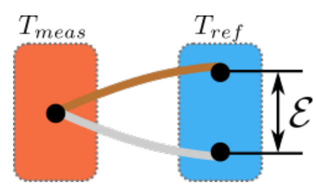
Consider a non-degenerate semiconductor at room temperature $T \ll \Delta$, in which the electron chemical potential μ is far away from the edges of the electron bands: $\mu \gg T$, $(\Delta - \mu) \gg T$.

(a) Assuming that the electron gas is ideal (non-interacting), calculate the densities of electrons $n_e = N_e^{\rm cond}/V$ in the conduction band and the number of holes $n_h = N_h^{\rm val}/V = (N - N_e^{\rm val})/V$ in the valence band as functions of temperature and the chemical potential, where N is the total number of electrons.

(b) Find the equilibrium chemical potential μ_i in the intrinsic semiconductor (i.e. without doping) and calculate the densities of holes and electrons $n_h = n_e = n_i$.

5.2 Thermocouple

Assuming the free ideal Fermi gas model for electrons in copper ($\varepsilon_F^{\rm Cu} = 7.0\,{\rm eV}$) and aluminum ($\varepsilon_F^{\rm Al} = 11.7\,{\rm eV}$), estimate the sensitivity



$$d\mathcal{E}/dT \approx \mathcal{E}/(T_{meas} - T_{ref})$$

of a Cu-Al thermocouple in $[V/K]$ for room temperatures T_{meas} and T_{ref}.

Hint: a temperature gradient in a metal leads to thermal emf because the electron chemical potential depends on temperature.

5.3 Degenerate relativistic Fermi gas

Consider an ideal Fermi gas of N spin-1/2 ultrarelativistic particles with energies $\varepsilon(p) = c|p|$ confined in volume V and at zero temperature. Calculate the energy and the pressure of the relativistic gas. What is the relation between the pressure, volume, and the energy?

Hint: start with computing the density of states $g(\varepsilon) = \frac{dN}{d\varepsilon}$ as a function of the number density $n = N/V$ and the Fermi energy ε_F. You may need it in the next problem.

5.4 Paramagnetism of a degenerate Fermi gas

Consider an ideal degenerate Fermi gas of spin-1/2 non-relativistic particles with magnetic moments $\mu_z = \pm\mu$, in a weak external magnetic field $H\hat{z}$, $\mu H \ll T$.

(a) Calculate the magnetic moment of the unit volume of the gas due to spin alignment/antialignment with respect to the magnetic field $H\hat{z}$.

(b) How does the result change if the gas is ultrarelativistic?

Hint: express the answer using the number density $n = N/V$ and ε_F.