PHY 540 Statistical Mechanics (Fall 2021) Homework 4
Posted: Thursday Sep 23
Due: Tuesday Oct 5

4.1 Gas leak hole

Molecules escape from a tank of gas into vacuum through a small hole of area A, $A \ll \lambda^2$, $A \ll V^{2/3}$, where λ is the mean free path in the gas and V is the volume of the tank. The gas is ideal with temperature T and the total number of molecules $N = nV$.
(a) Using the Maxwell's distribution, find the average energy of molecules in the jet
(b) Calculate (directly!) the thrust of the jet.
(c) Assuming that the gas is monoatomic and the leak is slow, find the equation for the change of the gas temperature.

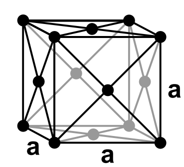
4.2 Chemical equilibrium

In a mixture of two monoatomic gases with number densities $n_{1,2}$ and molecular masses $m_{1,2}$, the molecules can react 1+2 $\to$ 3 to produce diatomic gas with molecular mass m_3 and binding energy $\epsilon_B = m_1 + m_2 - m_3 \ll m_{1,2,3}$. Find the density of the diatomic gas n_3 in chemical equilibrium. Assume that all the gases are ideal and the temperature T is such that $\hbar^2/(2I_3) \ll T \ll \hbar\omega_3$, where I_3 is the angular momentum and ω_3 is the frequency of vibrations of the diatomic gas.

4.3 Debye model of crystal vibrations

Use the tabular data below to estimate Debye temperatures for aluminum, copper, and lead. Note that these metals have face-centered cubic (FCC) crystal lattices, and an FCC lattice has 4 atoms per unit cell volume a^3, where a is the lattice spacing. Estimate contributions of lattice vibrations to the specific heat, express them in $[J/(g \cdot K)]$ and compare to reference values for these metals.

	Spacing a 10^{-10}[m]	At. weight μ g/mole	Long. sound u_L m/s	Transv. sound u_T m/s
Aluminum	4.05	26.98	6420	3040
Copper	3.62	63.59	4760	2325
Lead	4.95	207.2	2160	700

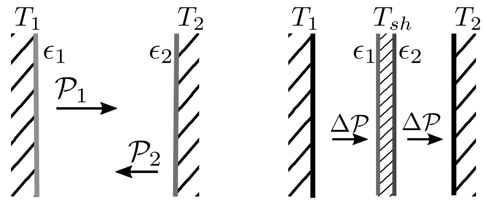


4.4 Radiation heat transfer

Emissivity $0 \le \epsilon \le 1$ is the fraction of radiation energy emitted by a unit of body's surface compared to the black body Stefan-Boltzmann's law,

$$\frac{1}{A}\frac{dE}{dt} = \epsilon\sigma T^4 \qquad (1)$$

where $\epsilon = 1$ corresponds to an ideal black body.
(a) Prove that such body's surface reflects $(1-\epsilon)$ fraction of the incident radiation.
(b) Find the rate of radiative heat transfer $\Delta\mathcal{P} = \mathcal{P}_1 - \mathcal{P}_2$ per unit area between two parallel surfaces with emissivities ϵ_1 and ϵ_2 and temperatures $T_1 > T_2$ (left figure).
(c) Now suppose that between the two surfaces there is a heat shield: a thermally insulated sheet that has emissivities ϵ_1 and ϵ_2 on its sides facing surfaces 1 and 2. Find the temperature of the shield T_{sh} and the heat transfer, assuming that the surfaces 1 and 2 are black (right figure).


T_1
T_2
ϵ_1
ϵ_2
$\mathcal{P}_1$
$\mathcal{P}_2$
T_1
T_{sh}
T_2
ϵ_1
ϵ_2
$\Delta\mathcal{P}$
$\Delta\mathcal{P}$