

3.1 Vertical column of ideal gas(25)

Use the Gibbs distribution over the single-molecule kinetic and potential energy, and integrate over $p_{x,y,z}$ and z to get the single-molecule partition function. Don't forget the Boltzmann factor for the partition function of the entire gas. For small height / large temperature approximation, expand in $mgh/T \ll 1$. The center of mass can be computed, e.g., by differentiating with respect to g .

3.2 Ultrarelativistic gas(25)

Treat the problem the same as the usual ideal gas except using the relativistic dispersion (kinetic energy vs. momentum) relation $\varepsilon = c|\vec{p}|$. Compute the partition function by integrating in spherical coordinates. Use TD relations to find the pressure as a function of temperature and volume. For adiabatic expansion, integrate the energy conservation (1st TD law) equation $dE = -p dV$.

3.3 Dipole molecules(25)

Use Gibbs distribution to average over the *solid angle* for all orientations of the polarization vector of a single molecule relative to the direction of the electric field. When computing the probability distribution $dp/d\theta$, do not forget the θ -dependency of the solid angle element $d\Omega/d\theta \neq \text{const}$. Compute the polarization by differentiating with respect to the electric field.

3.4 Paramagnetic medium II (25)

The dipoles are not interacting, and their partition functions are independent. Single-dipole partition function will yield a finite geometric series that can be reduced to a ratio of sinh functions. Compute all the quantities by differentiating the partition function Z or the free energy F .