PHY 540 Statistical Mechanics (Fall 2021) Homework 3
Posted: Thursday Sep 16
Due: Thursday Sep 23

3.1 Vertical column of ideal gas(25)

A column of ideal monoatomic gas of N molecules with height h_0 and surface area S is placed in the uniform gravity potential $U(h) = mgh$, where m is the molecule mass. Using the Gibbs distribution, find the total free energy F and the internal energy E (including the potential energy). Calculate the height of the center of mass of the column, and the heat capacity C. Consider separately two special cases $mgh_0 \gg T$ and $mgh_0 \ll T$.

3.2 Ultrarelativistic gas(25)

Using the Gibbs distribution, study an ultrarelativistic gas with dispersion relation $\epsilon(\vec{p}) = c|\vec{p}|$ in the Boltzmann limit
(a) Calculate its partition function and find its free energy, internal energy, entropy, and pressure.
(b) Find the equation of state and the change of pressure and temperature as functions of volume in adiabatic expansion.

3.3 Dipole molecules(25)

Consider an ensemble of independent classical diatomic molecules having permanent electric dipole moments b. The energy in an external electric field $\mathcal{E}$ is equal to $\epsilon = -\vec{b}\cdot\vec{\mathcal{E}}$. Calculate the partition function $Z_{1p}(T,\mathcal{E})$ and the angular probability distribution $dw/d\theta$ of molecules with respect to the electric field. Find the polarization $P(T,\mathcal{E})$, and calculate it in the limit of large temperature T. *(Consider only polarization energy in this problem, not kinetic energy of rotation or spatial motion.)*

3.4 Paramagnetic medium II (25)

Consider an ensemble of N non-interacting magnetic dipoles having spin J and quantized magnetic moments

$$\mu_z = \frac{m}{J}\mu\hat{z}\,, \quad \text{where } m_z = -J \leq m \leq +J\,. \tag{1}$$

The energy in an external magnetic field $\mathcal{H}\hat{z}$ is equal to $\epsilon = -\mathcal{H}\mu_z = -\frac{m}{J}\mu\mathcal{H}$.
(a) Calculate the partition function for N magnetic dipoles $Z_N(T,\mathcal{H})$, their free energy F_N and internal energy E_N.
(b) Calculate the total magnetization M, the susceptibility $\chi_T = \left(\frac{\partial M}{\partial \mathcal{H}}\right)_T$, and the dependence of magnetization on temperature $\left(\frac{\partial M}{\partial T}\right)_{\mathcal{H}}$.
(c) Calculate the heat capacity $C_{\mathcal{H}}$ at constant magnetic field $\mathcal{H}$.