2.1 Heat and work

Consider a sequence of infinitesimal Carnot cycles and integrate to the point then the two heat reservoirs have equal temperature.

2.2 Speed of sound

It is sufficient to consider one-dimensional wave propagation, in which the gas is locally displaced by a small $\xi(t,x) \ll \lambda$ (wave length) from its equilibrium. Thus, a thin slab of gas of thickness $L \ll \lambda$ is compressed by $\Delta L/L = \left(\frac{\partial \xi}{\partial x}\right)$, from which one can find the relative local change in density $\Delta\rho/\rho$ and the corresponding change in pressure $\Delta p \approx \left(\frac{\partial p}{\partial \rho}\right)\Delta\rho$. Use Newton's law to find the acceleration $\left(\frac{\partial^2 \xi}{\partial t^2}\right)$ and relate it to the spatial derivative $\left(\frac{\partial^2 \xi}{\partial x^2}\right)$, which will be the wave equation defining their phase velocity.

Use the VdW equation to find the relevant compressibility $\left(\frac{\partial p}{\partial \rho}\right)_S$ (explain why it has to be adiabatic with $S = \text{const}$).

2.3 Thermodynamics of an electric battery

(a) Motion of charge through the battery in the direction opposite to its polarity (i.e., current flowing into positive terminal) requires work proportional to the voltage and the charge, so it is natural to associate them with pressure and volume change of a gas, respectively.
(b) For a reversible process, the entropy is determined by the amount heat exchanged with the environment, $dS = \delta Q/T$. Use the energy conservation to relate the heat, the work of the battery on the charge, and change in the internal energy.
(c) Perform Legendre transform to obtain free energy as a function of temperature and stored charge. Consider the 2nd derivative of the free energy with respect to the temperature and the charge, and observe that it does not depend on the order of differentiation.
(d) Express derivative $\left(\frac{\partial U}{\partial q}\right)_T$ through $\left(\frac{\partial U}{\partial q}\right)_S$ and use the relation between derivatives from part(c).
(e) Depending on the sign of $\left(\frac{\partial E}{\partial T}\right)_q$, e.m.f. $\mathcal{E}(T)$ may be larger or smaller than $w(T)$. Hence, upon discharge the battery may need to be heated or cooled to stay at the same temperature.

2.4 Paramagnetic medium

(a) Since all spins are distinguishable (e.g., may be marked by their spatial position), they can be assigned to one of the three groups, spin-projection zero, spin-up, and spin-down. Compute the numbers of spins N_0, N_-, N_+ that satisfy the specified conditions. Compute the number of ways the spins can be distributed into these three groups (*multinomial theorem*) to obtain the statistical weight of such configurations and then the Boltzmann's formula for the entropy $S(x_0)$.
(b) The problem is posed as a microcanonical ensemble. In the true equilibrium state, the entropy S is maximal, which determines the value of x_0.
(c) Find temperature using definition $T = \left(\frac{\partial E}{\partial S}\right)_H$ and then solve for E and M.