PHY 540 Statistical Mechanics (Fall 2021) Homework 2
Posted: Wednesday Sep 8
Due: Thursday Sep 16

2.1 Heat and work

Suppose a working body has initial temperature T_{cold} and heat capacity C. How much work can be performed using this working body and an infinite heat bath at temperature T_{hot}? Calculate the maximal amount of work that can be extracted from 10 tons of ice at the melting point ($t_{\text{cold}} = 0^o\,\text{C} = 32\,\text{F}$) combined with an infinite heat bath at temperature $t_{\text{hot}} = 20^o\,\text{C} = 68\,\text{F}$. The latent heat of ice melting is $334\,J/g$ and the specific heat of water is $4.2\,J/(g\cdot K)$.

2.2 Speed of sound

Assuming that the compression is adiabatic and reversible in longitudinal sound waves, derive the speed of sound in gas

$$u^2 = \left(\frac{\partial p}{\partial \rho}\right)_S, \tag{1}$$

where $\rho = m_0 n = m_0/v$ is the mass density, n is the particle density and $v = 1/n$ is the volume per particle. Calculate the speed of sound in diatomic ($c_v = \frac{5}{2}$ per particle) ideal and van der Waals $[(p + \frac{a}{v^2})(v - b) = T]$ gases.

2.3 Thermodynamics of an electric battery

In this problem, thermodynamics of a chemical battery is studied analogously to a gas. If the battery is thermally insulated, its electromotive force is $\mathcal{E}(T)$ and depends only on its temperature. Neglect any changes in the battery's volume and Ohmic heat due to current, and assume that all charge/discharge processes are thermodynamically reversible.
(a) Identify equivalents of "pressure" and "volume" for the battery following the analogy with the work performed by expanding volume of gas.
(b) Write the thermodynamic identity connecting the changes of the internal energy dU and the entropy dS of the battery when charge dq flows through an external circuit.
(c) Introduce free energy of the battery - electric energy that can be extracted from the battery if its temperature is maintained constant. Write down relations between derivatives of the e.m.f. $\mathcal{E}$ and the entropy S.
(d) Now consider a battery in contact with a heat bath at constant temperature T. When charge dq flows through it, its internal energy changes by $(\Delta U)_T = -w(T)\Delta q$. Express $w(T)$ using $\mathcal{E}(T)$ and its derivatives (neglect the heat capacity of the battery).
(e) Assume that the emf $\mathcal{E}(T)$ decreases with increasing temperature. If the battery is thermally insulated, will its temperature increase or decrease under load (discharge)?

2.4 Paramagnetic medium

Consider a system of $N \gg 1$ non-interacting *distinguishable* magnetic dipoles. Each dipole can be in one of the three spin states, $\mu_z^{(i)} = 0, \pm\mu$. The energy of a single dipole in external magnetic field H is $\varepsilon^{(i)} = -H\mu_z^{(i)}$, and the total energy is

$$E = \sum_i \varepsilon^{(i)} = -H\sum_i \mu^{(i)} = -HM = -H\mu(N_+ - N_-)\,, \tag{2}$$

where M is the total magnetization and $N_\pm$ is the number of dipoles with spins $\pm\mu$.
(a) Compute the statistical weight and the entropy of the system if $N_0 = Nx_0$ is the number of dipoles with spin $\mu^{(i)} = 0$ and the total magnetization $M = \mu(N_+ - N_-) = \mu N\kappa$, where $\kappa = (N_+ - N_-)/N$.
(b) Find the equilibrium value of x_0 and the corresponding entropy $S(E, H)$ if E is the total energy

of spins in magnetic field H.
(c) Assuming $\kappa \ll 1$, calculate the temperature T and express E and M as functions of (T, H).