PHY 540 Statistical Mechanics (Fall 2021) Homework 10– **Optional**
Posted: Tuesday Nov 30
Due: Tuesday Dec 7

10.1 Master equation(25)

Consider Pauli's equation (also known as the kinetic balance or master equation)

$$\frac{\partial w_i}{\partial t} = \sum_j (P_{ij} w_j - P_{ji} w_i) \tag{1}$$

where $P_{ij} \geq 0$ is the rate of transition from state j to state i, and w_i are probabilities for the system states

$$w_i = \langle i|\rho|i\rangle \quad \Leftrightarrow \quad \rho = \sum_i |i\rangle\, w_i\, \langle i|$$

The rates are symmetric $P_{ij} = P_{ji}$.

(a) Prove that the normalization of probabilities $\sum_i w_i$ governed by equation (1) is conserved

(b) Prove that the entropy of the system described by equation (1) can only increase monotonically.

(c) Find the time-dependent probabilities $w_i(t)$ for a two-level system with $P_{12} = P_{21} = p$ and initial $w_1(t = 0) = 1$.

10.2 Frequency-dependent conductivity(25)

Use the relaxation time approximation to generalize the formula for the conductivity of a degenerate Fermi gas $\sigma(w = 0) = n_e q^2 \tau / m$ (Drude's formula) for the case of alternating current and voltage with frequency ω,

$$\sigma(\omega) = \frac{\sigma(\omega = 0)}{1 - i\omega\tau}\,. \tag{2}$$

Discuss the Joule heat generation in the limit of large frequency $\omega \gg \tau^{-1}$.

10.3 Transport phenomena in ideal gas(25)

Calculate the electric conductivity σ, the thermal conductivity κ, and the thermoelectric coefficient $\mathcal{S}$

$$\text{thermo e.m.f. } \frac{\partial V}{\partial x} = \mathcal{S}\frac{\partial T}{\partial x} \tag{3}$$

for a classical ideal gas of charged particles in relaxation time approximation with the relaxation time τ. Assume that the Coulomb interaction is completely screened, and the relaxation time is caused by short-range interactions.