PHY 540 Statistical Mechanics (Fall 2021) Homework 1
Posted: Wednesday Sep 1
Due: Thursday Sep 9

1.1 Entropy increase in irreversible processes

(a) Two equal amounts N_0 of diatomic gas (heat capacity $C_V = \frac{5}{2}N_0$) at temperatures T_0 and $3T_0$ are put into thermal contact. Calculate the change of entropy in each subvolume and the total when the thermal equilibrium is reached (volumes are constant, no particle exchange).
(b) Two equal volumes V_0 of ideal gas

$$PV = NT \tag{1}$$

at the same temperature but different pressures P_0 and $3P_0$ are separated by a partition. Calculate the change in entropy when the partition is removed (particles are indistinguishable).

1.2 Thermodynamic identities

(a) Calculate the following expression (*Hint: use Jacobian properties*) :

$$\left(\frac{\partial p}{\partial T}\right)_S \left(\frac{\partial V}{\partial S}\right)_T - \left(\frac{\partial p}{\partial S}\right)_T \left(\frac{\partial V}{\partial T}\right)_S$$

(b) Calculate $(\partial H/\partial p)_T$: express it using only equation of state, i.e. involving only p, V, T.
(c) How does the constant-pressure heat capacity C_p depend on the pressure, $\left(\frac{\partial C_p}{\partial p}\right)_T =?$
(d) Find general equation of state for a system in which the internal energy is independent of the volume and the enthalpy is independent of pressure

$$\left(\frac{\partial E}{\partial V}\right)_T = 0\,, \quad \left(\frac{\partial H}{\partial p}\right)_T = 0\,,$$

(use part (b) and a formula from lecture).

1.3 Thermodynamic potentials

A constant volume of gas has the heat capacity

$$C_V = aTe^{bT}$$

and the work done *by the gas* in isothermic expansion from V_1 to V_2 is

$$\Delta W = cT \log(V_2/V_1)$$

Find the equation of state and calculate the internal energy E, the free energy F, the enthalpy H, and the Gibbs potential G as functions of temperature and volume.

1.4 Van der Waals gas

Van der Waals gas has the following equation of state,

$$(p + \frac{a}{v^2})(v - b) = T$$

where v is the volume per particle, $v = V/N$. Assume that the c_V heat capacity is known.

- how does the internal energy depend on the volume at fixed temperature $\left(\frac{\partial E}{\partial V}\right)_T$?
- find how the temperature of the gas changes in unrestricted expansion into vacuum $V_1 \to V_2 > V_1$ (*hint: assume the gas does not perform work and does not exchange heat, so internal energy the E is constant, and you need to compute* $\left(\frac{\partial T}{\partial V}\right)_E$);